

Reconceptualizing Mathematical Problem-Solving Skills in The Digital Age: Teachers' Perspectives Through the Lens of Theory of Didactical Situations

Siti Ruqoyyah¹, Kana Hidayati¹, Jailani¹, Heri Retnawati¹, Wahyu Setyaningrum¹

¹ Universitas Negeri Yogyakarta, Indonesia

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ABSTRACT

Mathematical problem-solving skills are a critical 21st-century competency, yet their development still faces various challenges in the digital age. This study aims to explore teachers' perspectives on students' mathematical problem-solving skills, analyze teaching practices through the Theory of Didactical Situations framework, and formulate a reconceptualization of mathematical problem-solving skills relevant to the era of artificial intelligence. The study employed a qualitative approach with an exploratory design involving 11 junior high school mathematics teachers selected through purposive sampling. Data were collected via semi-structured interviews and analyzed using the Bogdan and Biklen model. Findings indicate that mathematics instruction remains dominated by a procedural approach that reinforces students' reliance on examples and teacher authority. Students struggle to understand non-routine problems, design independent strategies, engage in reflection, and verify solutions. Furthermore, the use of digital technology remains instrumental and does not yet support exploration or critical thinking. From a TDS perspective, the digital milieu has not yet functioned as an exploratory environment, while the didactic contract still positions the teacher as the primary authority. Based on a synthesis of empirical findings and studies on AI hallucination and Retrieval-Augmented Generation, this research proposes an expansion of the problem-solving framework by adding a dimension of digital information verification and formulating a conceptual model of the Didactic RAG-System to support mathematics learning in the era of artificial intelligence.

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Corresponding Author:

Siti Ruqoyyah

Universitas Negeri Yogyakarta, Indonesia; sitiruqoyyah.2025@student.uny.ac.id

1. INTRODUCTION

The digital age has fundamentally reshaped the way people acquire, process, and validate information, including in mathematics education (Drijvers & Sinclair, 2024; Engelbrecht & Borba, 2024). This transformation is marked not only by increased access to digital technology but also by a shift in the epistemological structure of learning, where knowledge no longer originates solely from teachers and textbooks but is disseminated through search engines, AI platforms, and various interactive digital environments (Pepin et al., 2017; Trouche et al., 2020). These developments are changing how students represent ideas, explore concepts, and solve mathematical problems (Jacinto & Carreira, 2023; Pittalis & Drijvers, 2023). Thus, mathematical problem-solving ability in the digital age can no longer be understood solely as a procedural skill, but also as the ability to navigate, evaluate, and verify the vast amount of information available in digital environments (Cevikbas et al., 2023; Weigand et al., 2024).

Mathematical problem-solving skills have long been regarded as a core competency in mathematics education. George Pólya's classical perspective views problem-solving as a process involving understanding the problem, planning a strategy, implementing a solution, and reflecting on the process (Polya, 2004; Schoenfeld, 2016). Over time, this ability has become an essential part of 21st-century competencies because it is closely linked to reasoning, critical thinking, and adaptive skills (Cai & Lester, 2010; OECD, 2023). PISA 2022 emphasizes that mathematical problem-solving is no longer merely symbolic manipulation, but rather the ability to use mathematics to solve complex contextual problems in real life (Ikenga & van der Sijde, 2024; OECD, 2019).

Nevertheless, various research findings indicate that students' mathematical problem-solving skills remain at a suboptimal level. Many students struggle with understanding the problem, selecting the appropriate strategy, engaging in reflective reasoning, and evaluating the validity of the solution (Amalina & Vidákovich, 2023; Ling & Mahmud, 2023). The 2022 PISA results indicate that the majority of students in developing countries, including Indonesia, still fall below the minimum level of reasoning-based mathematical competence (OECD, 2019). In Indonesia, although the Merdeka Curriculum has prioritized higher-order thinking skills, its implementation still faces challenges in meaningfully integrating digital technology (Gradini et al., 2025; Rizaldi & Fatimah, 2023).

Digital technology presents both new opportunities and challenges in mathematics education. Various digital platforms enable dynamic mathematical visualizations, interactive concept exploration, and access to a wide range of learning resources (Hillmayr et al., 2020; Hwang et al., 2023). However, the abundance of digital information also raises new issues regarding the validity of information and reliance on instant answers from technology (Kasneci et al., 2023; Zawacki-Richter et al., 2019). Furthermore, the phenomenon of AI hallucination, namely the tendency of AI to generate information that appears convincing but is incorrect makes the ability to verify information a critical component in solving mathematical problems in the digital age (Ji et al., 2023; Kasneci et al., 2023; Pardos & Bhandari, 2024).

These changes fundamentally shift the meaning of mathematical problem solving. The traditional problem-solving framework was developed in a stable learning context, when information sources were limited and teachers served as the primary authority on knowledge (Lester, 2013; Liljedahl et al., 2016). However, in the digital age, the problem-solving process takes place in a distributed knowledge environment, where students interact with various sources of information that may not necessarily be valid (Voogt & Roblin, 2012). Thus, mathematical problem-solving is no longer sufficiently understood as the ability to solve problems logically but also encompasses critical evaluation of the accuracy and validity of digital information (Kasneci et al., 2023; Voogt & Roblin, 2012). This situation highlights the need to reconceptualize mathematical problem-solving skills to make them more relevant to the characteristics of learning in the digital age.

This study employs the Theory of Didactical Situations (TDS) framework as its primary analytical tool. TDS views learning as a relational system involving interactions among teachers, students, and mathematical knowledge within didactic situations (Brousseau, 1997; Jessen et al., 2023). TDS has three main components: milieu (learning environment), didactic contract (implicit rules between teachers

and students), and didactic situation (self-directed learning (Novotná & Hošpesová, 2022). The digital age has transformed these three components: the milieu has expanded into the digital environment; the didactic contract has shifted due to the distributed nature of knowledge authority; and the didactic situation has become more complex due to students' direct interactions with AI (Clark-Wilson et al., 2020; Pepin et al., 2017; Trouche et al., 2020).

Research on mathematical problem-solving and digital technology still faces three major gaps. First, studies exploring teachers' perspectives on the changing nature of problem-solving in the digital age remain limited (Jiang et al., 2022; Stipek et al., 2001). Second, the use of the TDS framework to analyze AI-based learning remains rare, particularly in secondary schools (Jessen et al., 2023; Novotná & Hošpesová, 2022). Third, there has been little research reconceptualizing problem-solving skills by incorporating digital information verification as an integral component, especially given the phenomenon of AI hallucination (Ji et al., 2023; Rawte et al., 2023). The novelty of this study lies in the use of TDS to analyze teachers' perspectives and propose a reconceptualization by adding digital information verification as a new component, synthesized with reviews on AI hallucination and Retrieval-Augmented Generation (RAG) (Gao et al., 2023; Lewis et al., 2020).

Based on the above explanation, this study aims to: (1) explore teachers' perspectives on mathematical problem-solving skills in the digital age; (2) analyze mathematics teaching practices through the TDS framework, which encompasses the milieu, didactic contract, and didactic situation; and (3) formulate a reconceptualization of mathematical problem-solving skills relevant to the digital age as a theoretical contribution to the development of mathematics education in the era of artificial intelligence.

2. METHODS

This study employs a qualitative approach with an exploratory design to gain an in-depth understanding of teachers' perspectives on students' mathematical problem-solving skills in mathematics education in the digital age. The qualitative approach was chosen because this study focuses on uncovering teachers' meanings, experiences, and interpretations within a complex, dynamic, and contextual learning environment (Creswell & Poth, 2018). An exploratory design was used because the phenomenon under study still requires a deep initial understanding before it can be developed into a more structured intervention or conceptual model (Creswell & Creswell, 2018).

The study was conducted in five regions: Cimahi City, Bandung City, Bandung Regency, and West Bandung Regency in West Java Province, as well as Wonogiri Regency in Central Java Province. The selection of regions was done purposively, taking into account the diversity of geographical contexts, school characteristics, and access to digital technology. Regions in the Greater Bandung area were selected because they represent a spectrum of urban to semi-urban areas with relatively high levels of technology integration, while Wonogiri Regency was selected as a comparison region to obtain a variety of perspectives from different geographical contexts and educational infrastructure.

The study participants consisted of 11 junior high school mathematics teachers selected using purposive sampling based on the principle of information richness, meaning participants who were considered capable of providing in-depth information regarding the phenomenon under study (Patton, 2014). The number of participants was determined based on the principle of data saturation, which refers to the point at which additional interviews no longer yield significant new themes or information. Inclusion criteria included: (1) having at least two years of experience teaching mathematics, (2) currently teaching at the junior high school level, and (3) having experience using digital technology in mathematics instruction. Of the 11 participants, nine were female teachers and two were male teachers, with a regional distribution consisting of five teachers from Cimahi City, two from Bandung City, two from Bandung Regency, one from West Bandung Regency, and one from Wonogiri Regency.

The research focuses on three main aspects: mathematics learning conditions, students' mathematical problem-solving skills, and the use of digital technology in learning. These three aspects were analyzed using the Theory of Didactical Situations framework, which comprises three main

components: milieu, didactic contract, and didactic situation. The milieu was analyzed through the nature of students' interactions with their environment and digital learning resources. The didactic contract is analyzed through implicit rules, expectations, and patterns of interaction between teachers and students in the use of digital technology during mathematics learning. The didactic situation is analyzed through opportunities for students to construct knowledge independently through interaction with the digital environment without direct teacher intervention.

Research data were collected through online semi-structured interviews using an interview guide consisting of 22 questions. Semi-structured interviews were chosen because they allow for in-depth data collection while maintaining consistency across participants (Ruslin et al., 2022). All interviews were conducted via a video conferencing platform, lasted approximately 45–60 minutes, and were recorded with the participants' consent for transcription and data analysis. The following table shows the structure of the interview guide used.

Table 1. Structure of the Interview Guidelines

Domain	Number of Questions	Focus
Teaching Practices	7	Everyday mathematics learning and obstacles
Problem-Solving Skills	9	Student abilities based on teacher observations
Digital Technology	6	The use of technology and its challenges

Data analysis was conducted iteratively using a qualitative analysis approach based on the stages developed (Bogdan & Biklen, 2007), including: (1) verbatim data collection and transcription, (2) familiarization with the data through repeated reading, (3) data reduction, (4) coding, (5) categorization and identification of themes, and (6) interpretation and drawing of conclusions. The analysis process integrates inductive and deductive approaches. The inductive approach was used to identify themes emerging from the empirical data, while the deductive approach was used to interpret the data based on the Theory of Didactical Situations framework. The analysis process was conducted iteratively to ensure consistency in interpretation and depth of meaning.

The reconceptualization of mathematical problem-solving ability — the novelty of this study — was not derived directly from participants' responses but was constructed through a synthesis of empirical findings from interviews and a literature review on AI hallucinations and Retrieval-Augmented Generation (RAG). Thus, the resulting reconceptualization constitutes a conceptual contribution that requires further testing. Data validity was ensured through member checking (verification by 8 of the 11 teachers), peer debriefing (discussions with two senior researchers), and an audit trail (systematic documentation). All participants provided voluntary consent, and their identities were anonymized to maintain confidentiality.

3. FINDINGS AND DISCUSSION

The results and discussion are presented in an integrated manner following the stages of qualitative data analysis (Bogdan & Biklen, 2007), integrating an inductive approach to identify empirical themes and a deductive approach based on the Theory of Didactical Situations (TDS) framework. The analysis covers three main domains, namely mathematics learning conditions in junior high school, problem-solving ability profiles, and the utilization of digital technology, which are then synthesized into a reconceptualization of mathematical problem-solving ability in the digital age as the theoretical contribution of this study.

3.1. Conditions of Mathematics Learning in Junior High School

An analysis of seven interview questions regarding the conditions of mathematics learning in junior high school yielded nine themes that can be grouped into three main clusters: instructional design, student characteristics, and context and assessment. The following table presents the themes related to the conditions of mathematics learning in junior high school from the TDS perspective.

Table 2. Mathematics Learning Conditions in Junior High Schools from a TDS Perspective

Group	Main Theme	Implications in TDS
Instructional Design	A predominance of procedural approaches; the learning structure is organized but not yet exploratory	A strong didactic contract: the teacher as the primary authority on knowledge
Student Characteristics	Varied ability levels; weaknesses in basic numeracy; low mathematical aptitude	The learning environment has not yet accommodated the diversity in students' readiness to learn
Context & Assessment	Contextual learning efforts exist but are limited; assessment remains summative; time constraints and large class sizes	The didactic situation has not been optimally formed; the exploration process has not been evaluated.

Based on Table 2 above, the main finding indicates that mathematics instruction in junior high schools is still dominated by a lecture-based procedural approach. This is reflected in the teachers' statements.

"The most commonly used method is a lecture followed by a question-and-answer session, as it better ensures that the material is thoroughly covered."

Although teachers are familiar with various learning models such as Problem-Based Learning and Discovery Learning, their implementation remains limited and inconsistent due to considerations of time efficiency and student conditions. This situation aligns with the findings (Hiebert & Grouws, 2007) that the dominance of structured routines focused on content delivery rather than conceptual exploration remains a common characteristic of mathematics learning in the classroom. Teachers demonstrate adaptive flexibility, yet such adaptations have not yet moved beyond the framework of traditional approaches.

Another finding relates to the heterogeneity of student abilities. Students' abilities particularly weaknesses in basic numeracy and low mathematical disposition pose a significant structural challenge. The 2022 PISA results confirm that the majority of Indonesian students still fall below the minimum level of reasoning-based mathematical competence (OECD, 2023). Assessment, which remains dominated by summative assessments, combined with time constraints and large class sizes, further limits the scope for developing higher-order thinking skills (Clark-Wilson et al., 2020; Thurm & Barzel, 2022).

Within the TDS framework, these conditions reflect a strong didactic contract. This is evident in the teacher's role as the primary authority on knowledge, a learning environment that does not yet accommodate diverse levels of readiness, and a didactic situation that has not yet been optimally established (Artigue & Trouche, 2021; Brousseau, 1997). A fundamental shift is needed in instructional design to provide greater space for exploration, contextualization, and active knowledge construction by students (Drijvers & Sinclair, 2024; Weigand et al., 2024).

3.2. Profile of Students' Mathematical Problem-Solving Skills

An analysis of nine interview questions regarding students' mathematical problem-solving skills yielded six themes that were mapped to the four stages of problem-solving (Polya, 2004). In general,

students were able to solve routine problems identical to the teacher's examples, but experienced serious difficulties when faced with non-routine or contextual problems. The following are the findings regarding students' mathematical problem-solving ability profiles based on the teachers' perspectives and the TDS framework.

Table 3. Profile of Students' Mathematical Problem-Solving Skills Based on Teachers' Perspectives and the TDS Framework

Polya's Stages	Main Challenges (Teachers' Perspective)	Implications in TDS
Understanding the Problem	Failure to understand word problems; inability to identify key information; low mathematical literacy	The environment does not encourage critical reading of the issue
Designing a Strategy	Relies on the teacher's example; can only solve routine problems; uses trial and error without a plan	Didactic contract: students rely on the teacher's authority rather than their own reasoning
Implementing the Solution	Procedural errors; unsystematic approach; low accuracy	Suboptimal devolution; mechanistic learning
Review locking back	Not in the habit of checking answers; low metacognition and reflection	A non-didactic situation: there is no space for self-validation within the milieu

Findings from the problem-understanding stage indicate that students often fail to identify key information, particularly in word problems, as noted by the teacher:

"Students often don't understand the intent of the questions, especially word problems; they're unsure where to start."

Based on the findings, the ability to understand problems depends not only on procedural knowledge but also on metacognitive skills, which develop only through consistent exposure to non-routine problems (Schoenfeld, 2016).

Students' ability to develop strategies relies heavily on the teacher's examples and on mechanically following the same procedures. This is consistent with what the teacher stated:

"If the problem is similar to the example, I can solve it. But if it's even slightly different, I get confused right away and don't know what to do."

This reliance indicates that the ability to transfer knowledge and make mathematical connections has not yet developed. Procedure-based learning results in skills limited to the reproduction of algorithms, rather than flexible problem-solving (Niss & Højgaard, 2019). This also highlights how limitations in mathematical representation hinder conceptual understanding (Bouck et al., 2024).

The next stage is the solution implementation stage. During this stage, procedural errors, unsystematic steps, and low accuracy were observed. Meanwhile, during the review stage, students were not accustomed to evaluating their answers, reflecting low metacognitive and reflective abilities (Cai & Lester, 2010). In addition to cognitive factors, students' low mathematical dispositions also played a role. Low mathematical disposition is characterized by a lack of self-confidence, a tendency to give up easily, and minimal exposure to non-routine problems due to the dominance of traditional learning, all of which contribute to this condition (Borba et al., 2021; Hannula et al., 2004).

Within the TDS framework, this profile describes the limitations of a milieu that does not stimulate exploration, didactic contracts that foster dependence on the teacher, and the absence of optimal devolution, resulting in mechanistic learning (Ferretti & Gambini, 2025; Mangiante-Orsola et al., 2018).

Urgent improvements are necessary to optimize learning. Improvements need to be made in the design of a richer and more challenging environment, the formation of didactic contracts that encourage autonomy, and the creation of didactic situations that provide space for reflection and exploration (Bolstad, 2023; Lithner, 2017).

3.3. The Use of Technology in Education

An analysis of six interview questions regarding the use of digital technology yielded nine themes grouped into three dimensions: opportunities, limitations, and barriers. The findings indicate that although teachers are already using technology, the frequency and quality of its use still vary widely. Table 4 below presents the findings on the use of digital technology and their implications within the TDS framework.

Table 4. The Use of Digital Technology and Its Implications within the TDS Framework

Dimensions	Empirical Findings	Implications in TDS
Opportunities	GeoGebra and Quizizz have been introduced; they boost motivation and help visualize abstract concepts	The environment can be enriched through dynamic representations and interactive feedback
Limitations	Its use is still limited to presentations and assessments; it has not yet extended to exploration and problem-solving	The teaching contract remains unchanged: teachers remain the primary controllers of the learning process
Challenges	Limited facilities (projector, internet); teachers' digital skills are still lacking; limited preparation time	Such a non-instructional situation is rare because technology has not yet been used to encourage students' independent exploration

Based on the table above, in terms of opportunities, the use of platforms such as GeoGebra for geometry exploration and Quizizz for formative assessment demonstrates positive efforts toward interactive learning. Technology has been shown to increase motivation and aid in the visualization of abstract concepts (Mayer, 2020; Schindler et al., 2017). However, in terms of limitations, its use is still restricted to material presentation and simple assessments. A teacher remarked:

"It's more about engagement; we haven't reached the exploration stage yet."

This situation indicates that technology integration is still in the substitution phase, not the transformation phase (Engelbrecht & Borba, 2024). Infrastructure barriers including a shortage of projectors and laptops, unstable internet connections, and a lack of teachers' digital competencies and limited preparation time further complicate these transformation efforts (Li et al., 2025; Tondeur et al., 2019).

Within the TDS framework, this situation can be analyzed through three main components. First, the learning environment has the potential to be enriched through dynamic representations and interactive feedback; however, current technology does not yet function as a system that challenges students to think (Clark-Wilson et al., 2020; Drijvers & Sinclair, 2024). Second, the didactic contract has not undergone significant changes despite the use of technology. Teachers remain the primary controllers of the learning process, so technology merely replicates traditional practices in a digital format (digitizing traditional teaching) (Engelbrecht & Borba, 2024; Thurm & Barzel, 2022). Third, didactic situations have become rare because technology is not utilized to encourage students' independent exploration, even though interactive digital environments have been shown to improve problem-solving quality when students are given autonomy to explore and make decisions (Pittalis &

Drijvers, 2023; Tamborg, 2021). Thus, the integration of technology requires a more fundamental pedagogical transformation, not merely the addition of digital tools within an existing learning framework.

This situation is becoming increasingly critical given the proliferation of generative AI among students, where the phenomenon of AI hallucination the tendency of AI systems to generate information that appears convincing but is factually inaccurate is potentially exacerbated by the absence of verification guidance from teachers (Ji et al., 2023; Kasneci et al., 2023; Pardos & Bhandari, 2024). When technology is used without a pedagogical framework that encourages critical evaluation, students become vulnerable to accepting AI output as truth without an adequate verification process. It is this challenge that guides this study toward a reconceptualization of mathematical problem-solving ability, as outlined in the following section.

3.4. Reconceptualizing Mathematical Problem-Solving Skills in the Digital Age

Empirical findings indicate that students are not yet accustomed to reflecting on and evaluating the solutions they arrive at. Furthermore, findings regarding the still largely instrumental use of technology suggest that digital integration in mathematics education has not yet fully supported the development of students' reflective and evaluative skills. This situation becomes increasingly complex in the era of generative AI, when students can obtain instant answers from various digital systems that may not necessarily be valid. This situation indicates that problem-solving skills in the AI era can no longer be understood merely as procedural skills, but also as the ability to evaluate the validity of digital information used during the problem-solving process. Meanwhile, the development of generative AI and the phenomenon of AI hallucination (Ji et al., 2023; Kasneci et al., 2023; Pardos & Bhandari, 2024) point to the need for information verification skills. The combination of limitations in classroom technology practices and the complexity of AI-based environments drives this research to explore Retrieval-Augmented Generation (RAG) as a conceptual lens for reformulating mathematical problem-solving skills in the digital age.

The synthesis of the three empirical findings above with the literature review on AI hallucinations and Retrieval-Augmented Generation (RAG) led to a reconceptualization of mathematical problem-solving abilities, which constitutes the main theoretical contribution of this study. Empirically, it was found that: (1) learning remains procedure-dominated, thereby limiting exploration and critical reasoning; (2) students are not accustomed to verifying the accuracy of solutions and evaluating information sources; and (3) technology is used instrumentally without fostering critical thinking. This situation is exacerbated by the proliferation of generative AI systems that allow students to obtain instant answers without understanding the underlying thought process. Even more critically, the phenomenon of AI hallucination the tendency of large language models to generate information that appears convincing but is factually inaccurate—makes the ability to verify information a new critical component in contemporary mathematical problem-solving (Ji et al., 2023; Kasneci et al., 2023; Pardos & Bhandari, 2024; Rawte et al., 2023).

The traditional problem-solving framework (Polya, 2004) was developed in a relatively stable learning context with limited information sources and the teacher as the sole authority. In the digital age, the problem-solving process takes place in a distributed knowledge environment where students interact with various information sources whose validity is not guaranteed. This reconceptualization expands Polya's framework by adding a fifth dimension: the ability to verify digital information that is, the capacity to evaluate the accuracy, relevance, and validity of information, including AI outputs used throughout the problem-solving process (Bransford et al., 2000; Lester, 2013; Voogt & Roblin, 2012).

This expansion of the problem-solving framework is operationalized through the conceptual model of the Didactic RAG-System, which synthesizes the principles of Retrieval-Augmented Generation with the Theory of Didactical Situations framework. RAG is an AI architecture that integrates retrieval (searching for information from verified databases), augmentation (context

enrichment), and generation (producing valid context-based solutions) to generate responses that are more factually reliable (Gao et al., 2023; Lewis et al., 2020; Zhao et al., 2026). This model illustrates the integration of the AI environment, the processes of information retrieval and verification, the expansion of mathematical problem-solving stages, and TDS components in mathematics learning in the era of artificial intelligence. This conceptual integration is visualized more systematically in Figure 1.

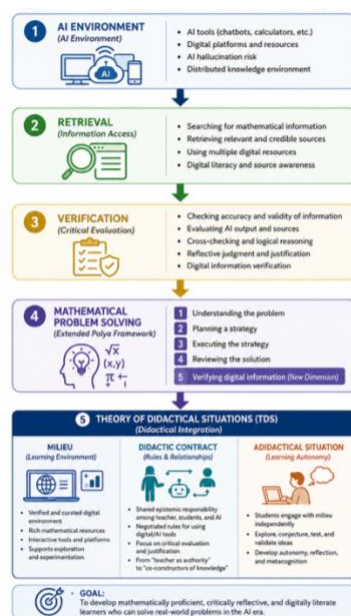


Figure 1. Conceptual Framework of Didactic RAG-System

Based on Figure 1, the AI environment functions as a digital milieu that provides access to various sources of mathematical information. In this model, the retrieval process allows students to access information from relevant and verified sources, while the verification process positions the critical evaluation of AI output as an integral part of mathematical problem-solving. These processes are then integrated with an extension of Polya’s framework through the addition of a dimension for verifying digital information. Furthermore, the entire process is interpreted within the TDS framework, which encompasses the milieu, the didactic contract, and the adidactic situation. The relationship between the RAG mechanism, the extension of the stages of mathematical problem solving, and the TDS components in the Didactic RAG-System model is explained in more detail in Table 5.

Table 5. Conceptual Model of the Didactic RAG System: Integration of RAG, Polya’s Stages, and TDS Components

Components	RAG Mechanism	Polya's Stages	TDS Components
Source Verification	Retrieval: search from a verified database	Understanding the Problem	Verified environment
Contextual Enrichment	Augmentation: integration of relevant context	Designing a Strategy	New didactic contract
Solution Development	Generation: generating valid context-based solutions	Implementing the Solution	Digital adidactic situation
Critical Evaluation	Validation: Students evaluate the accuracy of the AI's output	Review locking back	New epistemic contract

In the Didactic RAG-System model, AI does not replace the teacher's role but functions as a structured and verified component of the learning environment. At the environmental level, the system provides reliable epistemological feedback so that students do not merely receive answers but interact with validated sources of knowledge. At the didactic contract level, this model encourages the formation of a new epistemic contract in which the responsibility for verifying the truth of information is shared among the teacher, the student, and the AI system, shifting students from the position of passive recipients to active evaluators of the mathematical information used. At the didactic situation level, the system provides space for students to interact directly with verified knowledge sources independently, without having to rely on the teacher's sole authority (Artigue & Trouche, 2021; Brousseau, 1997; Novotná & Hošpesová, 2022).

Theoretically, this conceptual model expands the framework of mathematical problem solving to make it more relevant to the characteristics of learning in the era of artificial intelligence, while also offering a conceptual integration between TDS and AI architecture—a topic that remains underdiscussed in the mathematics education literature. Practically, this model provides guidance for teachers in designing learning situations that position information verification as a meaningful mathematical activity so that students can develop the epistemological literacy necessary to learn critically and responsibly within an AI ecosystem (Kasneci et al., 2023; Weigand et al., 2024).

Thus, the third objective of this study has been achieved at the conceptual level through the formulation of the Didactic RAG-System model as an integrative framework between the Theory of Didactical Situations and Retrieval-Augmented Generation for understanding mathematical problem-solving abilities in the era of artificial intelligence. As a conceptual contribution, this model still requires further empirical testing through the implementation of AI-based didactic situations in mathematics education.

4. CONCLUSION

This study indicates that students' mathematical problem-solving skills in the digital age still face various limitations. Mathematics instruction in junior high school remains dominated by a procedural approach that reinforces students' reliance on examples and the teacher's authority. As a result, students struggle to understand non-routine problems, devise strategies independently, engage in reflection, and verify the correctness of their solutions. Furthermore, the use of digital technology in learning remains instrumental in nature and does not yet fully support exploration or critical thinking.

From the perspective of the Theory of Didactical Situations, these findings indicate that the digital environment has not yet functioned as an exploratory setting that supports independent knowledge construction, while the didactic contract still positions the teacher as the primary authority in learning. Therefore, the transformation of mathematics education in the digital age requires more fundamental pedagogical changes than merely the integration of technology.

As a theoretical contribution, this study proposes an extension of the mathematical problem-solving framework by incorporating the dimension of digital information verification as an integral part of the problem-solving process. Based on a synthesis of empirical findings and a literature review on AI hallucinations and Retrieval-Augmented Generation, this study also formulates a conceptual model of the Didactic RAG-System as an integration between TDS and AI architecture to support more reflective, critical, and adaptive mathematics learning within the AI ecosystem.

This study is still limited to a conceptual exploration based on teachers' perspectives; therefore, the resulting model requires further empirical testing through the implementation of AI-based learning in the context of mathematics education.

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